Neural, Parallel, and Scientific Computations 25 (2017) 403-416

MODELING HARMFUL ALGAL BLOOMS IN THE WESTERN BASIN OF LAKE ERIE AND AN ECONOMIC SOLUTION

MARK MILLER[1] AND HEM RAJ JOSHI[2]

[1] BrownFlynn, Cleveland 44113, USA

[2] Department of Mathematics, Xavier University, Cincinnati OH 45207-4441, USA

ABSTRACT.

In 2014, the city of Toledo, Ohio was without drinking water for approximately three days due to the a harmful algal bloom lingering around the city's water intake. While many factors played a role in permitting a bloom of this magnitude, one of the most dominant factors has been the introduction of the Zebra Mussel in 1990. Its arrival heralded yet another dramatic change in the trophic interactions of the Great Lakes, namely due to its enormous consumption of algae, its primary food source. This led to a "Diatom abundance [decline of algae]"82 - 91 %" and a 100% increase in water clarity, the first major locus of the invasion.

We propose a simple relationship between algal species and Zebra Mussel through a basic ecological predator-prey model: the Lotka-Volterra model. Through this model, we can learn what level of Zebra Mussel population will maximize water transparency through the elimination of algal populations with the most important result being the elimination of large-scale algal blooms. Then, the economic impact of the bloom will be assessed, as well as the necessary decrease in phosphorus loading needed to sufficiently reduce the risk of compromising the water supply.

AMS (MOS) Subject Classification. 35E99, 93A30, 97M10.

1. Introduction

In August of 2014, the city of Toledo, Ohio (population approximately 500,000) was without drinking water for roughly three days due to a harmful algal bloom of *Microcystis aeruginosa* (a single-celled blue green alga, or cyanobacterium) lingering around the city's water intake. *Microcystis aeruginosa* only emits toxins under certain conditions, but as it would turn out, Lake Erie is an ideal environment for growing said toxic strains [1]. While many factors played a role in permitting a bloom of this magnitude, one of the most dominant factors has been the introduction of the Zebra Mussel *(Dreissena polymorpha).* The invasive species was introduced around 1988 to the Great Lakes via ballast of barges [2]. Ballast, or stabilizing water for barges, comes from nearly all corners of the earth. The Zebra Mussell likely arrived in Eastern European ballast and was able to outcompete local mussels due to its phenomenal rate of growth [3]. An adult is able to produce up to 1 million oocytes

Received January 23, 2017

in a single year [4]. Its arrival heralded yet another dramatic change in the trophic interactions of the Great Lakes, namely due to its enormous consumption of algae, its primary food source [5]. This led to a Diatom abundance decline of “82-91%” and a 100% increase in water clarity in Lake Erie, the first major locus of the invasion [6].

The explanation for the rising domination of algal species is less clear, although recent findings are startling. The Maumee River watershed, the largest watershed to enter the Great Lakes [7], also happens to be one of the most heavily cultivated watersheds in the Great Lakes. This watershed was once covered by the Great Black Swamp which occupied land in Ohio, Indiana, and Michigan. Prior to the clearing of the watershed, the water running into the Maumee river estuary was clear and virtually devoid of sediment or phosphorus from fertilizer. This is no longer the case, and in 2006 year nearly 2,000 metric tons of phosphorus ran into the lake–a roughly average figure for the past decade. With the rise of phosphorus loading came the rise of algal blooms which have been present in Western basin since the 1960s [8].

Extensive study has been conducted on the impacts of various natural parameters on the production of *Microcystis aeruginosa* and Zebra Mussels such as wind, temperature, water clarity, and nutrient availability. Other studies have even documented the predator-prey relationship between these two species, including the efficiency of Zebra Mussel predation upon algal species. However, no work appears to exist on how artificially modifying parameters that drive this predator-prey relationship (through ecological engineering) may lead to the collapse or suppression of *Microcystis aeruginosa* through Zebra mussel predation.

It is the combined proliferation of these two invasive species that has given rise to the assessment below. While both species in question are detrimental to the health of the lake, perhaps it is possible to harness the relationship that they share in order to exploit it to the advantage of society. A decline in the Zebra Mussel population would be desirable, but a decline in *Microcystis aeruginosa* populations would be the more desirable outcome by an enormous margin. Therefore, the purpose of this paper is two-fold: Establish the mathematical predator-prey relationship between algal species and Zebra Mussels, and examine the relationship for potential leverage with particular attention to the extirpation of algal blooms.

In section 2, we discuss the models selected and, in section 3, the parameters used in these models. We illustrate numerical simulations in section 4, and section 5 is devoted to discussion. We discuss model failures and future considerations in section 6 and cost shifting as a solution is discussed in section 7.

2. **Original Models and Model Adjustments**

The original Lotka-Volterra equations are given by:

$$\frac{dA}{dt} = r_1 A(t) - \alpha_1 A(t) M(t) \tag{2.1}$$

$$\frac{dM}{dt} = -r_2 M(t) + \alpha_2 A(t) M(t) \tag{2.2}$$

where $A(t) and M(t)$ represent algae and mussel populations, respectively. Note that for each species there are two terms, one negative and another positive (for a total of four). These two terms reflect the species' ability to grow (the positive term) and its inability to survive or escape the predator (the negative term). The term with more weight (which is determined by the coefficients to be discussed) determines whether the species' population will expand or contract.

However, these equations have two fundamental failures. The first is that there is no carrying capacity for the prey in the absence of the predator. For example, suppose that the system contained no Zebra Mussels. According to the original Lotka-Volterra equations, the population of the algal species would expand indefinitely. In reality, this is never observed. At a certain population, the prey population depletes essential resources such as their own food supply or even space. This is fixed by adding the logistic term as in (2.3). As the population of prey increases, the logistic term goes to zero thereby eliminating the growth term. Also, mathematicians and biologists alike realized that it was unrealistic for the consumption to increase in proportion with the rise in population of the predation. Therefore, Holling introduced the functional response curve to the predator equation [9].

Now we modify the Lotka-Volterra equations by introducing carrying capacity and functional response:

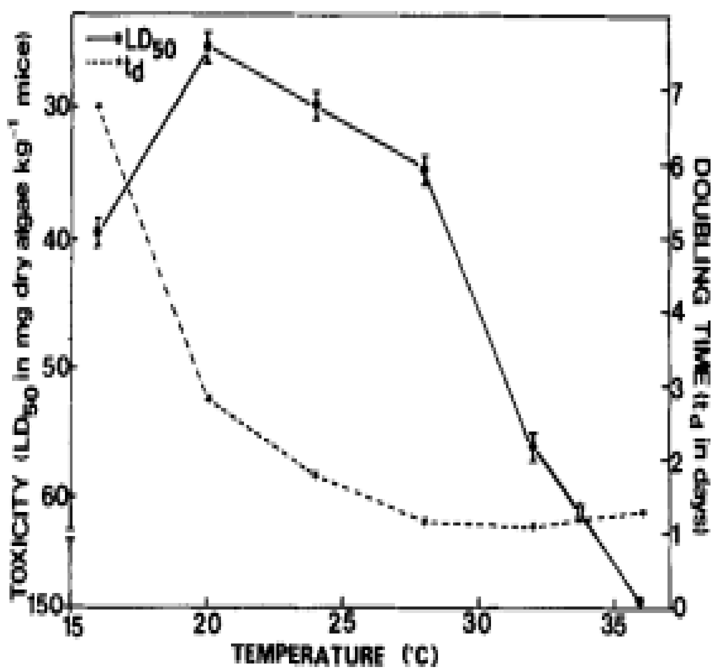
$$\frac{dA}{dt} = r_1 \left(1 - \frac{A(t)}{K_1}\right) A(t) - \frac{\alpha_1 A(t) M(t)}{1 + \beta A(t)} \tag{2.3}$$

$$\frac{dM}{dt} = -r_2 M(t) + \frac{\alpha_2 A(t) M(t)}{1 + \beta A(t)} \tag{2.4}$$

The significance of the variables in these equations will be described in the following section.

3. **Parameters Used in Model**

r_1**: Growth rate in the absence of predation.** This parameter describes how many individuals are produced per individual per day. In these models, r_1 is measured in Diatom/[(Diatom)(Day)] [10]. Though the value of r_1 varies considerably and is dependent on a number of factors, for our purposes it is set to 0.217. This value is specific to Lake Erie. To arrive at this value, we found a temperature index created by Westhuizen et al. [1]. Their paper provides growth rates as a function of temperature.



From this graph we can see that the average summer temperature of Lake Erie (18.9 °C) is almost exactly the same as the 20 °C at which the researchers found the greatest production of toxins—a doubling rate of 3.2. This is a high replication rate, something that we would expect in a small organism. The temperature and phosphoric loading have a combined condition quality exceeded by few locations in the U.S. Using this doubling time, we calculate the growth rate using the following equation:

$$r_1 = \frac{ln(2)}{t_{double}} \tag{3.1}$$

$$r_1 = \frac{ln(2)}{3.2} \approx 0.217 \tag{3.2}$$

r_2**: Death rate in the absence of prey.** This parameter describes how fast the predator starves in the absence of prey. The mortality rate for the Zebra Mussel, r_2, is a poorly researched parameter. This presents a real problem. In attempt to overcome this lack of data, we have varied the parameter to hopefully capture the effect of the different death rates under most conditions. Through running many iterations of the model, the value 0.15 presented itself as the most viable value; it is the value we have chosen to use in our model simulations. For example, a model that used too large of a value for r_2 would never permit the Zebra Mussel to establish a colony, which is not at all the case in Lake Erie which teams with colonies. On the other hand, if the value was too small, the population of the Zebra Mussels would continuously exceed the population of the algal species which is also unlikely.

α_1: **Predation coefficient.** This parameter measures capture efficiency, or the effect of predator per capita growth rate [10]. This value also has little research associated with it, but 0.01 is a value used by ecologist Michael Bulmer for invertebrate interactions [9]. In some of the sample model simulations, the value of α_1 has been changed to highlight its importance and effect in the model.

α_2: **Energy conversion coefficient.** This parameter describes the efficiency of the predator at assimilating energy. Once gain, the species-specific value is poorly researched, so Bulmer's parameter of 0.1 in his invertebrate model will be used [9].

β: **Functional response curve coefficient.** This curve accounts for predator satiation. Bulmer's value of 0.005 from his invertebrate model will be used.

K_1: **Carrying capacity of prey.** This parameter describes the maximum number of individuals allowed in the prey population.

4. Simulations

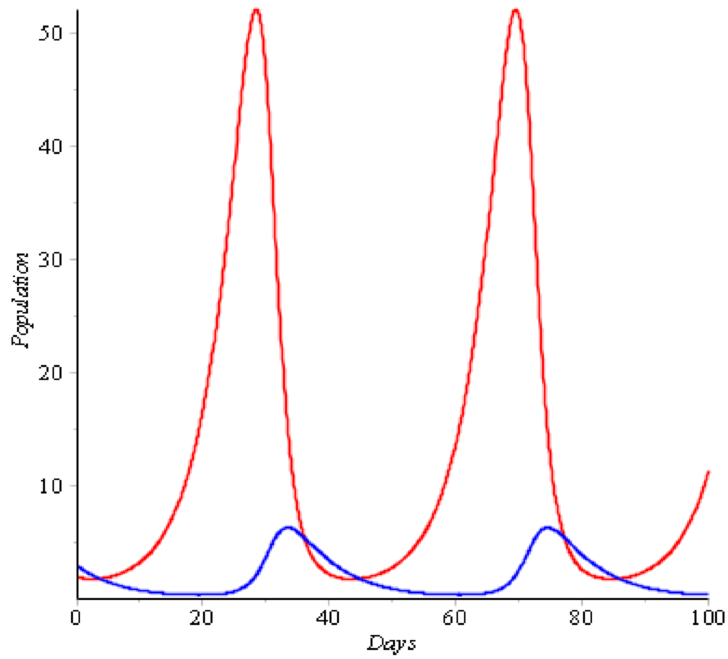
Three sample results have been selected. The selection of models was based on several conditions:

1. Its ability to reflect general, basic qualities which we know exist in Lake Erie. For example, we know that at almost any given time, there are many times more algal individuals than mussel individuals. We know algal species grow faster, and that they are more sensitive to environmental changes. Thus, the model selection can be described as a heuristic process whereby models that fit the demands of reality are then varied to see what possibilities there are within these heuristic constraints. In the process of research, hundreds of iterations of the model were run, but only a small percentage of the iterations had variations on the parameters that were remotely plausible.

2. The model's ability to demonstrate the effect of altering a parameter. Within the small percentage of models that made it past the basic screening, an even smaller percentage of results were different with limited inspection. It is from this pool that we have drawn the sample models.

Simulation 1: Lotka-Volterra without carrying capacity and functional response

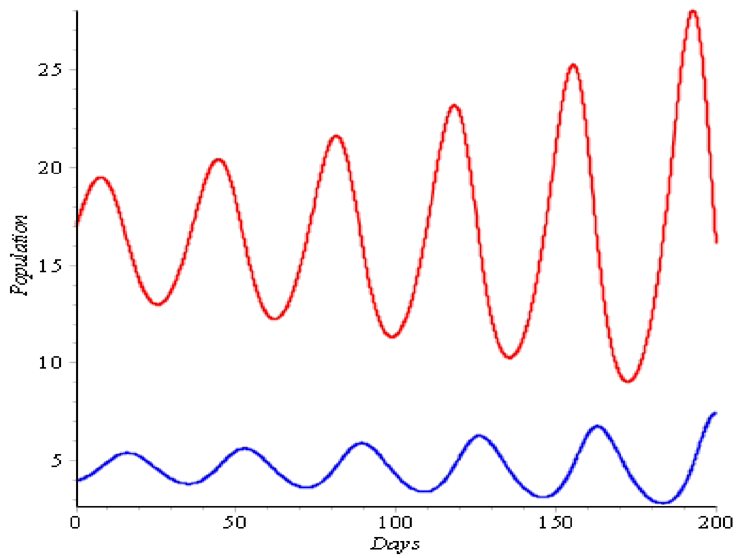
This first model is an example of the Lotka-Volterra model without the adjustments that were made by Hollings and others over time. We set the parameters to $r_1 = 0.217, r_2 = 0.15, \alpha_1 = 0.1$ and $\alpha_2 = 0.01$ in equations (2.1) and (2.2). The initial algal population is 2 and the initial mussel population is 3. Note the population scale for mussels and algae are not the same. We have chosen not the set the scale, but we can assume that the scale for the mussels is standard (1 unit = 1 mussel), while the units for the algae is in the millions or billions. The algal species is represented in dark color and the mussels are represented in light color.



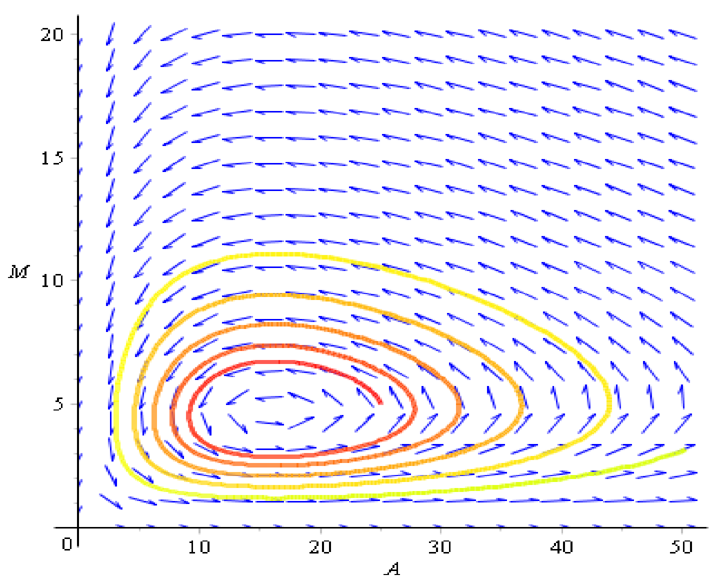
Note how quickly the algal species grows in the first 30 days of the simulation. Around 30 days, we begin to see a strong response from the mussel population. This causes the algal population to crash. There is even a brief period where the algal species' population is exceeded by the population of the mussels. This is not unrealistic. In fact, inverted biomass pyramids are quite common in aquatic systems [11]. In the ocean, the vast reproductive capabilities of phytoplankton are exploited by organisms higher up on the food chain at such a fast rate that the biomass of the phytoplankton does not exceed the biomass of the predating organisms. The cycle seems to continue on with a certain level of stability.

Simulation 2: Lotka-Volterra with carrying capacity and functional response

Here we look at a simulation that incorporates the functional response and carrying capacity discussed earlier. We set the parameters to $r_1 = 0.217, r_2 = 0.15, \alpha_1 = 0.05$, $\alpha_2 = 0.01$, $\beta = 0.005$, and $K_1 = 1000$ in equations (2.3) and (2.4). The initial algal population is 17 and the initial mussel population is 3. Note that the value for α_1 has been made half as large as in the previous model. Since this is the predation efficiency coefficient, the end result is that fewer algae individuals should be consumed.



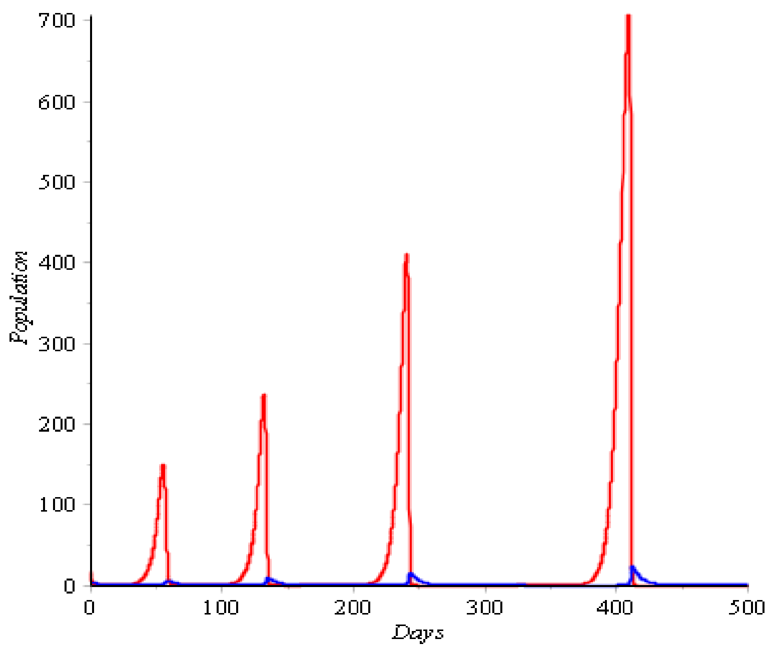
As suggested by the smaller value for α_1, the population of the algal species does not respond as strongly to the predation of the mussels. Additionally, we see that the mussel population and algal species do not overlap as they did in the first simulation. This is due in part to the satiation that was introduced by the functional response curve. If the curve were to be removed from the equation, we would find that the oscillation of the mussel's population curve would have a larger amplitude. Finally, it would appear that the population of each species will get progressively larger, but we know this to not be the case because the carrying capacity of the system has been set to 1000 algal units.



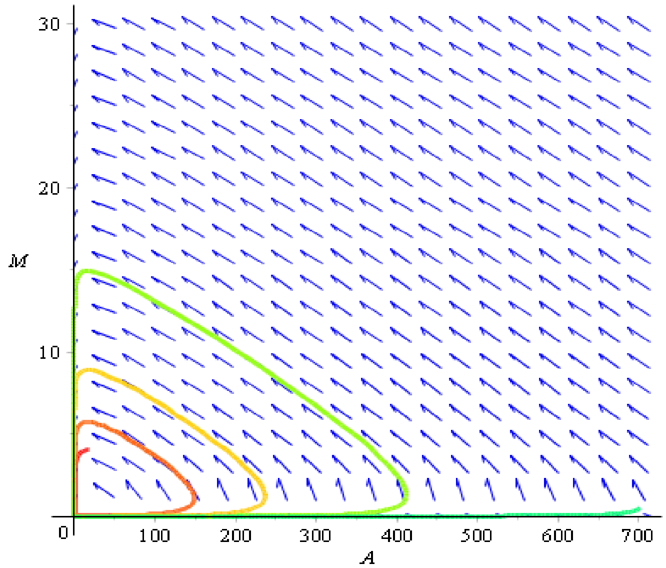
A nice visual aid for the relationship between the species can be found in the vector field of the same solution. Again, no matter what point is selected on the vector field, one would be inclined to think that the expanding spiral would continue indefinitely, but we know from the carrying capacity that this cannot be the case.

Simulation 3: Lotka-Volterra with carrying capacity, functional response, and high predation

Here we look at an example where the efficiency of the predator is extremely high. We set the parameters to $r_1 = 0.217, r_2 = 0.15, \alpha_1 = 0.3$, $\alpha_2 = 0.01$, $\beta = 0.005$, and $K_1 = 1000$ in equations (2.3) and (2.4).



Because the efficiency of the predator was set to a relatively high value, the crash of the algal species (in dark color) is more pronounced. Each time the algal bloom emerges, it persists for only a small period before the response of a relatively small population of mussels brings the bloom to halt. It should be noted that $\alpha_2 = 0.01$ has not changed. This means that despite the increased predation by the mussels, the mussel's actual ability to take the harvested biomass and convert it into other mussel individuals has not improved. This is similar to the efficiency of a cow. Cows have a high level of predation upon grass, but limited ability to assimilate that biomass because of the difficulty of digesting it.



The vector field for the same system of equations draws out the extreme nature of the relationship established here. Note the scale on each axis. The algal species is only able to proliferate in the near-absence of the mussel. As with the prior simulation, the oscillation will be limited by the carrying capacity, though that constraint is not depicted due to the scale of the system of equations.

5. **Discussion**

Now that the first goal of the project–namely the establishment of a relationship of the two species–has been set it in place, it is worth analyzing the results to accomplish the second objective of the project which is to examine the relationship for potential leverage. The first main result that is suggested by the models is that both species are here to stay. In none of the scenarios did the Zebra Mussel or algal species go extinct. This result matches the findings of empirical data; while a number of lakes have reported enormous die-offs of both algal blooms and Zebra Mussels (a phenomenon that is far more natural for algal blooms), never has there been a case in which Zebra Mussels have been extirpated from the lake. This demonstrates that no program should ever expect to make Zebra Mussels extinct, but rather expect to severely damage the population. To this end, experimenting with various Lotka-Volterra models suggests that there are three means by which control of algal blooms by Zebra Mussels is realistic.

The first is by maximizing the predation coefficient. As seen in the third model, boosting that parameter in any way spells the end of any serious algal bloom. For example, boosting calcium in the lake would lead to increased shell length and thus increased efficiency per mussel unit. Calcium clearly has a direct effect on the ability of Mussels to both consume and grow. Zebra Mussels are almost entirely absent from Lake Superior due to the lack of dissolved calcium in the water[12]. Spraying has proven to be effective with another invasive species called Sea Lampreys. A certain compound is regularly sprayed at their main spawning grounds in the St. Mary's River and has proven effective in controlling their abundance. Of course the spraying would have a different purpose entirely if done with calcium. However, as with any modifications to complex aquatic systems, intense ecological impact analyses should take place before such drastic measures are ever taken.

Another more difficult means of decreasing algal blooms (and their toxicity) would be to control the warming of the lake. At first consideration, this may seem an outlandish suggestion. However, the reality is that the average temperature of the lake has steadily been rising over the years due to increased sedimentation and climate change. Sedimentation darkens the lake, allowing it to absorb more sunlight. The next section of this paper outlines an action plan for addressing sedimentation (and the phosphoric loading that accompanies it).

Finally, increasing the energy conversion of the Zebra Mussels is a potential means for increased control of algal blooms. Studies have shown that the energy conversion of the Zebra Mussel is much lower for toxic species such as *Microcystis aeruginosa* [13]. Leveraging the competitive nature of the algal species in Lake Erie to ensure that a less toxic strain dominates the annual blooming season would lead to increased energy conversion because the mussels would be able to assimilate more material in the absence of toxins. Unlike the first two proposals, however, the means for specifically weakening the status of *Microcystis aeruginosa* in Lake Erie are unknown.

6. Model Failure and Future Considerations

Below are a list of various factors that the model (Lotka-Volterra system of equations) used in this context fails to address.

- The rate of replication for *Microcystis aeruginosa* varied dramatically depending on the temperature.
- The population of *Microcystis aeruginosa* is not only dependent on the phosphoric loading from the Maumee River watershed, but also up the species that compete with it. All other algal species demand phosphorus.
- Over the course of the past several decade, *Microcystis aeruginosa*, which had originally been almost completely absent from the lake, has begun to rise in population. Now, it is perhaps the most dominant species in the lake. Taking into account the evolutionary components would be difficult.
- Wind plays an important role in the growth of the algae. When concentrations are high and wind is low, there is decreased local food availability. Accounting for wind is not possible in these models.

- Turbulence plays an important role in the growth of *Microcystis aeruginosa*. Research suggests that one of the reasons that *Microcystis aeruginosa* has become so abundant in the Western Basin is due to a swimming bladder that the species has that other algal species do not. This gives *Microcystis aeruginosa* a competitive advantage.
- Zebra mussels are generalists and feed on nearly any type of benthic, organic material.
- Zebra mussels are prey to certain species of diving ducks. How can the models account for predation of the predator?
- There is an assumed uniformity among the performance of the predator, but age of the mussel can play a significant role in this.

7. Cost-shifting as a Solution to Lake Erie's Algal Blooms

In earlier part of this paper, the relationship between Zebra Mussels and algal blooms has been discussed in detail. However, at the base of this predator-prey relationship lies an essential phenomenon: nonpoint source pollution. It is nonpoint source pollution that is responsible for the size of Lake Erie's algal blooms today. While tampering with the ecosystem could yield promising results, the true battle takes place in the fields. To defeat algal blooms almost certainly involves reducing nonpoint sources of phosphorus, of which agriculture is the primary perpetrator. However, controlling this runoff is both unpopular and expensive. On the other hand, the costs of the practice are also unpopular and expensive. Not only that, but there are significant externalities that are involved in the costs of nonpoint source pollution which are impossible to quantify. This leaves the community with two fundamental options:

1. Pay the expenses of dredging the Maumee River tributary annually
2. Pay the expenses associated with intercepting nutrient-laden water before it ever reaches the tributary

The tragedy of the commons (the minimal marginal cost of adding one field into the phosphoric sinking capacity of the watershed) long ago gave rise to the selection of the first choice. But now that the capacity of the lake is being pushed to the limits, the community is beginning to see the error of its ways.

Rather than suggest policy prescription, tax programs, or something akin to that, perhaps a more elegant solution is in the community's midst. The most ideal solution would have the following qualities:

1. Economic feasibility
2. Intergenerational sustainability

Laying out the qualities desired in the solution before it is found helps to ensure that whatever solution is settled upon does not eventually deteriorate into the same issues experienced before.

There is a path to both of these qualities. Firstly, there are strong costs associated with the phosphoric and sediment loading which are quantifiable to the public (in USD). Secondly, there is a thing such as a solution that not only does not deteriorate with time, but appreciates in value: green infrastructure.

The costs associated with phosphoric loading are numerous, varied, and present a good opportunity to build a portfolio of cost structures which will soon prove to be essential to a reasonable solution. An easily quantifiable cost is the municipal budget of $5 million annually applied to dredging the estuary of the Maumee River. As the water comes down in unnaturally large torrents thanks to an efficient tiling and ditch system, sediment and phosphorus find their way out of the field and settle in the estuary. It is estimated that over 50% of Lake Erie's sediment load and 40% of the phosphorous load is contributed by Maumee River watershed [14]. Green infrastructure (the kind that consists of land and vegetation) presents a beautiful solution to issue of the maintenance needed to maintain traditional grey infrastructure as green infrastructure will, once in place, only increase in value. The organisms that compose green infrastructure only get larger and sequester more carbon as time goes on; the organisms only become more efficient at straining water; and the organisms only prevent more runoff and erosion.

7.1. **Outline of Solution.** The most central notion to the proposed solution below is cost-shifting: preventing runoff from the entering the main watershed and capturing the resulting savings to invest in projects that will eventually mitigate or eliminate a majority of the costs incurred to society due to negative downstream impacts. The city would begin by approaching investors and offering them the opportunity to invest in building a series of strategically mapped marshes (a form of green infrastructure) throughout the watershed to intercept phosphorous- and sediment- laden runoff. The investors would them be repaid using a standard compounding interest model that draws funds from the saved money set aside for annual dredging or any other avoided costs.

Given that the Maumee River watershed has set costs associated with nonpoint source pollution, if we simply shift the costs from addressing the symptoms (via dredging) to curing the problem (via the construction of upstream marshes), little extra capital need be expended. It should be noted that we are not considering any externalities, therefore the return on the shift in cost is expected to be significant.

Below is a list of processes and designs that may be undertaken to execute the program described above. Those implementing the program can decide what would be the most chronologically responsible manner to proceed.

1. Identifying cost structures
2. Acquiring investors
3. Acquiring municipal momentum to permit the cost-shifting
4. Calculating the rate of return and run time of program
5. Finding farmers willing to participate in the program and land eligibility
6. Marsh positioning
7. Marsh design
8. Cost estimation and contracting
9. Land management
10. Temporal considerations

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