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RELIABILITY OF GENERAL SERIES-PARALLEL AND SEQUENTIAL SERIES-PARALLEL SYSTEMS AND THEIR OPTIMIZATION

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ABSTRACT. In this paper we derive the explicit formula for system reliability of sequential series-parallel structure by using recursive method and we also obtain the optimal number of redundant components of the system of general series- parallel system and seqeuenting series-parallel system under the consideration of cost minimization. Each component failure follows Poisson probability distribution and inter-failure time distibution follows exponential distibution.The comparative study of both types of reliability configuration is carried out from the cost point of view. To show the applicability of the reliability models under study the numerical illustrations have also been obtained.

Key Words: Series-parallel, Reliability, Optimal, Redundant

AMS (MOS) Subject Classification.62B05, 34E15.

1. Introduction

Reliability is the probability that a system will perform its intended work satisfactorily for a specified operating condition. Reliability of complex system is studied under various structural framework such as series structures, parallel structures, series-parallel structures, parallel-series structures, sequential structures, combined structures. Series system is characterized by the property that the failure of one of these components results the failure of whole system. Reliability of series system can be enhanced by making provision of redundant components in the system. The study of optimization of reliability of complex system, their analysis and evaluation have been getting the attention of several researchers. It is worthnoting to mention some of the contributions. Djerdjour and Rekab (2001) presented a nonlinear integer programming model and they alsomade the evaluation of reliability of series- parallel system. Ruan and Sun (2006) derived exact solution for cost minimization by using greedy algorithm. Mustafa (2017)proposed three methods - reduction metod, hot method and cold method for improving the reliability of a series-parallel system.

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He also obtained mean time to failure(MTTF) for each proposed methods. Xiao et al. (2010) proposed sensitivity analysis method based on P-boxes interval algorithm and linear aggression analysis. Chun et al.(2015) developed Chun-song-paulino(CSP) method to compute parameter sensitivities of system failure probability by using sequential compounding method. Lee et al. (2003) made comprehensive study of two methods max-min method and the method proposed by Nakagawa and Nakishima for the reliability evaluation of series-parallel system. Vargas et al.(1991) obtained the exact solution of cost minimization of series-parallel system with multiple component choices using an algebraic method. El. Said and Raghab (2016) studied the avaibility and relaibility of the of the preventive maintence model with two types of failures (type 1 and type 2) by using supplementary techniques and Laplce transforms.

Redundancy is a common approach to improve the reliability and availability of a system. Adding redundancy increases the cost and complexity of a system in retern of the high reliability. If cost does not play vital role, use of redundant components may be the attractive options such as in the manufacture of machinery of aero plane, equipments of nuclear power plants. Use of redundant component obviously increase the reliability but motto of the scientist is to design the structure of redundant components so as to minimize the number of components and cost ultimately. Many researchershave been engaging in modeling the structure of redundant components of complex system. Tian et al. (2009) applied a practical approach for joint reliability redundancy optimization of multistate series-parallel systems. Liang and Smith (2004) solved redundancy allocation problem by using an ant colony meta-heuristic optimization method. Yalaoui et al. (2005) investigated the problem as an linear integer programming in redundancy allocation in a parallel-series system reliabilty. Linmin (2015) introduced the availability equivalence factor method to compare different system designs wherein two types of availability equivalence factors of the repairable multi-state series-parallel system are derived. Dhillon (1994) made the study of income optimization analysis of a repairable system composed of two units in parallel with one warm standby. Tian et al.(2008) developed recursive algorithms for efficient performance evaluations of the multi-state k-out-of-n system. Wang et al.(2013) investigated a multi-state Markov repairable system with redundancies to evaluate the reliability. Li and Zuo (2008) presented a recursive algorithms for reliability evaluation of multi-state weighted k-out-of-n systems. Hu et al. (2016) extended the concept of availablity equivalance from general binary repairable system to discrete multistate parallel-series system by using universal generating function technique.

In this paper we set up the system of equations and derive the explicit formula for the system reliability of sequential series-parallel structure by using recursive method and we also obtain optimal number of redudant components of the system of general series-parallel system and seqeuencing series-parallel system under the consideration of cost minimization. The comparative study of both types of reliability configurations is also carried out from the cost point of view.

2. **Mathematical model**

We derive the explicit formula forsystem reliability of sequential series-parallel system. The notations which we use in our model are.
m: number of subsystem.
n_i: number of components used in i-th subsystem,
$i = 1, 2, ..., m$.
R_{i_j}: reliability of the j-component for the i-th subsystem,$i = 1, ..., m, j = 1, ..., n_i$.
R_0: admissible level of reliability of the whole system.
x_{i_j}: number of redundant of j-th components used in i-th subsystem, $i = 1, ..., m$,
$j = 1, ..., n_i$
$x^*_{i_j}$: optimal number of j-th redundant components used in i-th subsystem,
$i = 1, ..., m, j = 1, ..., n_i$
c_{i_j}: cost of j-th component for i-th sub system,$i = 1, ..., m, j = 1, ..., n_i$
u_{i_j}: upper bounds of number of j-th components for the i-th subsystem,$i = 1, ..., m$,
$j = 1, ..., n_i$
λ_{i_j}: rate of falure of j-th component for the i-th subsystem,$i = 1, ..., m, j = 1, ..., n_i$
In our model, we make the following assumptions:

- Components have two states: working or failed.
- The reliability of each component is known and is deterministic.
- Failure of individual components are independent and identically distributed.
- Failed components do not damage other components or the system, and they are non repairable.
- Each component failure follows Poisson probability distribution and inter-failure time distibution follows exponential distibution.

2.1. **Reliability of general series-parallel system.** A series-parallel system consists of m disjoint subsystem that are connected in series and subsystem$i$ $(1 \leq i \leq m)$ consists of n_i components that are connected in parallel. The reliability block diagram of a series parallel system has been shown in Figure 1. In such a series parallel system, there are m minimal cuts and they do not have any components in common.The reliability of this system can be obtained by:

$$R_{sp} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n_i}(1 - R_{i_j})]. \tag{2.1}$$

$$R_{sp} = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n_i}(1 - e^{-\lambda_{i_j}t})].$$

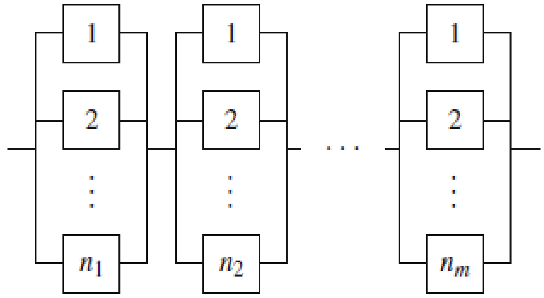

FIGURE 1. series parallel system

2.2. **Reliability of Perfect Sensing and Switching Mechanism.** The sensing and switching mechanism is said to be perfect if it is instantaneous and failure free, when the mechanism is perfect a standby component is switched into operation as soon as the active component becomes failed. The system fails when the last component fails in active operation. This can be illustrated by figure 2 that, the system with three cold standby components is working under sensing mechanism provision. When component 1 fails, the sensor (ss) switches immediately to the component 2 and then to component 3 when component 2 fails.

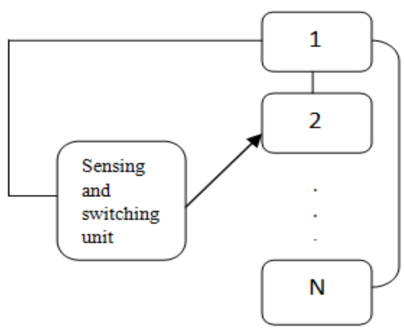

FIGURE 2. sequensing standby system

For a cold standby system with n different components.
when $n = 1$, $R_1(t) = e^{-\lambda_1 t}$.

when $n = 2$, $R_2(t) = \frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_2 t} = [\sum_{j=1}^{2} e^{-\lambda_j t} \prod_{i\neq j} \frac{\lambda_i}{\lambda_i-\lambda_j}]$.

when $n = 3$, $R_3(t) = \frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_2 t} - \frac{\lambda_1\lambda_2[(\lambda_2-\lambda_3)e^{-\lambda_1 t}+(\lambda_3-\lambda_1)e^{-\lambda_2 t}+(\lambda_1-\lambda_2)e^{-\lambda_3 t}]}{(\lambda_1-\lambda_2)(\lambda_2-\lambda_3)(\lambda_3-\lambda_1)}$.

$$= [\sum_{j=1}^{3} e^{-\lambda_j t} \prod_{i\neq j} \frac{\lambda_i}{\lambda_i-\lambda_j}].$$

Therefore in general the system reliability for n number of components in sequential standby system is

$$R_s(t) = \sum_{j=1}^{n} e^{-\lambda_j t} \prod_{i\neq j} \frac{\lambda_i}{\lambda_i - \lambda_j}. \tag{2.2}$$

2.3. **Sequential series-parallel system.** A sequential series-parallel system consists of m disjoint subsystem that are connected in series and subsystem i $(1 \leq i \leq m)$ consists of n_i components that are connected in parallel. The reliability block diagram of a serie-parallel sequensing system is given in figure 3. In such a sequential series-parallel system, there are m minimal cuts and they do not have any components in common.

For i=1 and $n_1 = 1$,

$$R_s(t) = e^{-\lambda_{1_1} t} = [\textstyle\sum_{j=1}^{n_1=1} e^{-\lambda_{1_j} t} \prod_{\alpha \neq j} \frac{\lambda_{1_\alpha}}{\lambda_{1_\alpha} - \lambda_{1_j}}].$$

For i=2 and $n_1 = 1, n_2 = 2$

$$R_s(t) = [e^{-\lambda_{2_1} t}[\tfrac{\lambda_{2_2}}{\lambda_{2_2} - \lambda_{2_1}}] + e^{-\lambda_{2_2} t}[\tfrac{\lambda_{2_1}}{\lambda_{2_1} - \lambda_{2_2}}]] * e^{-\lambda_{1_1} t}$$

$$= [\textstyle\sum_{j=1}^{n_2=2} e^{-\lambda_{2_j} t} \prod_{\alpha \neq j} \frac{\lambda_{2_\alpha}}{\lambda_{2_\alpha} - \lambda_{2_j}}] * e^{-\lambda_{1_1} t}$$

$$= \textstyle\prod_{i=1}^{2} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}}.$$

For i=3 and $n_1 = 1, n_2 = 2, n_3 = 3$

$$R_s(t) = [e^{-\lambda_{3_1} t}[\tfrac{\lambda_{3_2}}{\lambda_{3_2} - \lambda_{3_1}} \tfrac{\lambda_{3_3}}{\lambda_{3_3} - \lambda_{3_1}}] + e^{-\lambda_{3_2} t}[\tfrac{\lambda_{3_1}}{\lambda_{3_1} - \lambda_{3_2}} \tfrac{\lambda_{3_3}}{\lambda_{3_3} - \lambda_{3_2}}] + e^{-\lambda_{3_3} t}[\tfrac{\lambda_{3_1}}{\lambda_{3_1} - \lambda_{3_3}} \tfrac{\lambda_{3_2}}{\lambda_{3_2} - \lambda_{3_3}}]] * [e^{-\lambda_{2_1} t}[\tfrac{\lambda_{2_2}}{\lambda_{2_2} - \lambda_{2_1}}]$$

$$+ e^{-\lambda_{2_2} t}[\tfrac{\lambda_{2_1}}{\lambda_{2_1} - \lambda_{2_2}}]] * e^{-\lambda_{1_1} t}$$

$$= [\textstyle\sum_{j=1}^{n_3=3} e^{-\lambda_{3_j} t} \prod_{\alpha \neq j} \frac{\lambda_{3_\alpha}}{\lambda_{3_\alpha} - \lambda_{3_j}}] * [\sum_{j=1}^{n_2=2} e^{-\lambda_{2_j} t} \prod_{\alpha \neq j} \frac{\lambda_{2_\alpha}}{\lambda_{2_\alpha} - \lambda_{2_j}}] * e^{-\lambda_{1_1} t}$$

$$= \textstyle\prod_{i=1}^{3} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}}.$$

Therefore in general for m subsystem and n_i components in sequential series-parallel system, the system reliability is

$$R_s t = \prod_{i=1}^{m} \sum_{j=1}^{n_i} e^{-\lambda_{i_j} t} \prod_{\alpha \neq j} \frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}}. \tag{2.3}$$

where $i = 1, 2, 3, ..., m,;$ $j = 1, 2, 3, ..., n_i;$ $\alpha \neq j; \alpha \in j.$

2.4. **Optimization of the model.** The optimization problem of series-parallel system whose general formula has been derived as equation (2.1) can be expressed as:

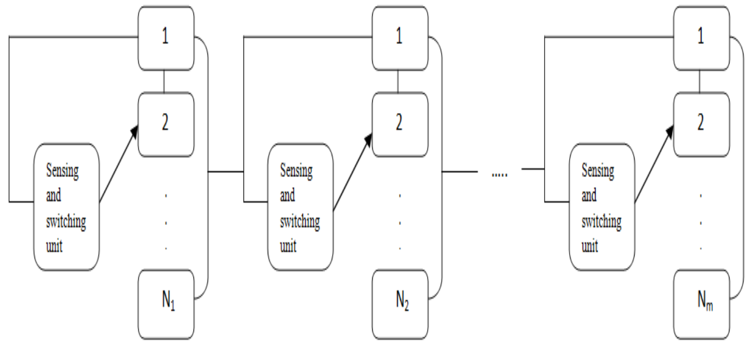

FIGURE 3. Sequential series - parallel system standby system

$$\text{minimize}\, C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{i_j} * (x_{i_j}) \tag{2.4}$$

$$\text{subject to the constraints}\, R_s(t) = \prod_{i=1}^{m}[1 - \prod_{j=1}^{n_i}(1 - R_{i_j})^{x_{i_j}}] \geq R_0$$

$$x \in X = (x|0 \leq x_{i_j} \leq u_{i_j}, i = 1, 2, ..., m, j = 1, 2, .., n_i).$$

this problem consists of $m * n_i$ variables and $m * n_i + 1$ constraints.
Also, the optimization problem of sequential series-parallel system whose general formula has been derived as equation (2.3) can also be expressed as:

$$\text{minimize}\, C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{i_j} * (x_{i_j}) \tag{2.5}$$

$$\text{subject to the constraints}\, R_s(t) = \prod_{i=1}^{m}\sum_{j=1}^{n_i}(R_{i_j})^{x_{i_j}}(\prod_{\alpha \neq j}\frac{\lambda_{i_\alpha}}{\lambda_{i_\alpha} - \lambda_{i_j}}) \geq R_0$$

$$x \in X = (x|0 \leq x_{i_j} \leq u_{i_j}, i = 1, 2, ..., m, j = 1, 2, .., n_i).$$

3. Numerical results and interpretations

(i) To compute the system reliability, we use the values of following parameters in equations (2.1) and (2.2), $m = 1$ and $n = 1$ and $c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3$ then the graph given by equation(2.1) and (2.2) is shown in figure 4. Graph explores that for single component system, the reliabilities remain the same for both the structures, sequential series-parallel system and general series-parallel configurations which is quite natural.

(ii) If $m = 3, n_1 = 1, n_2 = 2$ and $n_3 = 3$ and $c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} =$

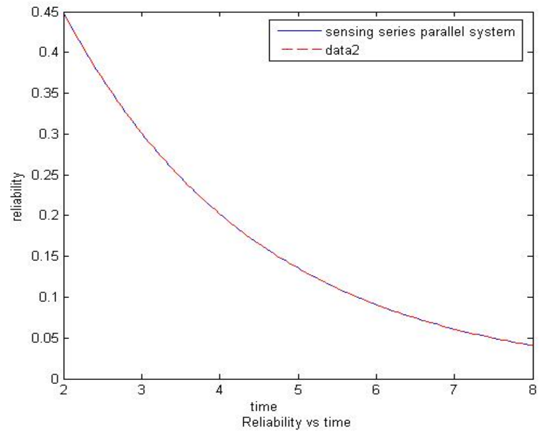

FIGURE 4. graph of reliability vs time

$3; c_{3_3} = 5; \lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31; t = 3$ using the same above equations(2.1)and (2.2), the graph is shown in figure 5.

Graph explains that sequential series-parallel system has higher reliability than that

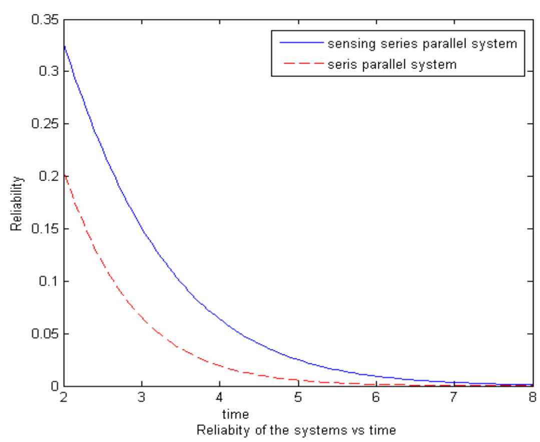

FIGURE 5. graph of reliability vs time

of general series-parallel system.

(iii). Table 1 is the optimal solution table of general series-parallel system obtained by using minimization problem represented by(2.4).This table represents the optimal number of redundent components used in the subsystem for the following parameters as the known constants. $c_{1_1} = 2; c_{2_1} = 3; c_{2_2} = 4; c_{3_1} = 2; c_{3_2} = 3; c_{3_3} = 5$, $\lambda_{1_1} = 0.1; \lambda_{2_1} = 0.2; \lambda_{2_2} = 0.3; \lambda_{3_1} = 0.11; \lambda_{3_2} = 0.21; \lambda_{3_3} = 0.31$, time $t = 3$, $u_{1_1} = 5; u_{2_1} = 6; u_{2_2} = 7; u_{3_1} = 8; u_{3_2} = 9; u_{3_3} = 10; R0 = 0.9$.

ov	X11	X21	X22	X31	X32	X33
46.2611	3.7617	0.2198	0.5203	0.1125	9.5213	1.4416
38.3979	1.5775	0.8557	1.7006	2.6964	1.1927	3.3805
40.8899	3.9996	1.4555	2.4431	3.5837	2.0076	1.1123
37.7292	6.3822	0.0023	3.3563	2.7510	0.4453	0.9390
40.0898	2.6590	0.6108	3.3277	2.9242	4.0248	0.3411
34.1067	1.2377	0.9608	5.5147	0.0652	1.2569	0.5578
34.4025	3.1647	1.9242	2.3317	2.1984	0.7944	1.2387
37.5462	1.7079	2.0323	0.9468	0.3867	3.9336	2.3344
42.3928	0.8843	5.8209	0.2110	0.0472	5.5409	1.1201
51.1166	0.4176	0.5704	0.5902	4.7350	3.8403	5.0437
46.0402	4.0704	1.9048	2.4611	9.3165	1.0679	0.1008
27.2893	3.1290	1.7399	1.0116	0.2902	3.6055	0.0737
44.1769	1.0804	1.1860	1.4662	1.425	5.3696	2.7269
27.1657	5.3686	1.7138	0.033	3.4728	1.3012	0.0611
40.5405	4.8424	0.2733	2.157	1.4775	0.8291	3.1926
47.5564	4.4598	5.5917	3.3391	3.2874	0.0054	0.3828
30.8343	5.2670	0.0722	1.6998	0.0272	3.0167	0.8361
20.4360	1.3419	0.1245	1.2953	1.6686	2.6025	0.2105
42.9488	0.5704	4.9795	2.9370	0.6128	2.1027	1.5175
19.0703	1.0095	1.1185	0.1306	4.0371	1.2874	0.2474
52.1194	2.0040	5.2186	1.3688	4.5438	0.4352	3.3174

Table 1,Optimal solution table with objective functions of series- parallel system

This table explains that minimization objective function value is the least 19 monetary unit for the optimal number of redundant components $x^*_{1_1} = 1, x^*_{2_1} = 1, x^*_{2_2} = 0, x^*_{3_1} = 4, x^*_{3_2} = 1, x^*_{3_3} = 0$.

(iv). Table 2 is the optimal solution table of sequential series-parallel system obtained by using minimization problem represented by (2.5). This table gives the optimal number of redundent components used in the subsystem by taking the same known constant parameters as given for the table 1.

ov	X11	X21	X22	X31	X32	X33
8.1558	0.3889	0.1641	0.0229	3.1007	0.0717	0.0755
11.3478	0.4906	1.0873	0.6890	1.4988	0.2449	0.1233
14.6098	1.4958	0.4493	0.3177	1.6581	1.7241	0.1022
10.6537	1.0357	0.4009	0.2860	1.7173	0.3350	0.3592
10.6715	0.2266	1.3577	0.7145	0.1680	0.8285	0.0932
12.1885	2.0001	0.5052	0.2966	0.0729	1.5002	0.1680
13.0242	0.2522	0.0901	1.7690	0.1277	1.6049	0.0206
5.0559	0.5886	0.0699	0.1021	0.5434	0.6559	0.0412
13.6288	0.2156	0.1835	0.4183	2.5252	1.3396	0.3810
13.2558	2.1561	0.5467	0.2973	0.4978	0.3182	0.8329
11.2164	1.1155	0.1659	0.1778	1.4078	0.4829	0.7024
11.2295	0.1041	0.9148	0.1123	2.1518	0.1840	0.5945
14.4785	2.5259	0.2604	0.3037	1.0557	0.7967	0.5858
12.6107	0.8015	0.8817	0.7735	0.0596	0.6123	0.6625
12.9255	2.2910	0.4698	1.3831	0.3175	0.2346	0.0125
15.8497	3.0951	0.1918	1.2136	0.1206	0.4876	0.5051
10.8146	0.1277	0.2246	1.8961	0.2371	0.4207	0.1129
11.0046	0.5807	0.4845	1.0514	0.4270	0.1057	0.6026
10.5409	0.1386	0.3781	0.9163	1.6681	0.6721	0.0223
12.2999	0.5240	0.8184	0.3654	0.8148	0.2172	1.0108
9.9156	0.5668	0.5174	0.8592	0.2593	0.7524	0.2035

Table 2 Optimal solution table with objective functions of sequential series-parallel system

This table reveals that minimization objective function has the least value 5 monetary unit for which optimal number of redundant components are $x^*_{1_1} = 1, x^*_{2_1} = 0, x^*_{2_2} = 0, x^*_{3_1} = 1, x^*_{3_2} = 1, x^*_{3_3} = 0$.

In the comparision of table 1 and table 2, we find that the sequential series-parallel system (table 2) appears more economical than the general series-parallel system (table 1) so as to maintain the relaibility standard 0.9 for both the systems.

4. **Discussion and Conclusion**

We have derived the explicit formula for system reliability of sequential series-parallel system and we have also obtained the optimal number of redundant components of general series-parallel system and seqeuential series-parallel system under the consideration of cost minimization. Comparative study made on general series-parallel system and sequential series-parallel system enable us to understand that sequential series-parallel system is more economical and more reliable than the general series-parallel system. The reliability and optimization of series-parallel system and sequential series-parallel system may have ubiquitous applications in nuclear power plants, aero space industries, telecomunications, complex manufacturing systems, textiles industries and machinery systems.

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