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A NOTE ON THE NEW PHAM'S SOFTWARE RELIABILITY MODEL

ANGEL GOLEV[1], GEORGI SPASOV[2], AND MARTIN STIEGER[3]

[1,2]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA
[3]Allensbach University
Lohnerhofstrasse 2, D-78467 Konstanz, GERMANY

ABSTRACT: In [1] Pham considered a new two–parameter lifetime distribution and generated a new mean value function $m(t)$ that represents the expected number of software failures to be detected by the time t considering the uncertainty of operating environments by:

$$m(t) = N\left(1 - \frac{\beta}{\beta + at\left(\frac{t}{2}\ln(bt) - \frac{t}{4} + \frac{1}{b}\right)}\right)^{\alpha}$$

where α, β, a, $b > 0$.

The determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the Hausdorff approximation, i.e. the " saturation " in Hausdorff sense of the function $m(t)$ to the horizontal asymptote - the subject of study in the present paper.

In this regard, we will make some comparisons between the new Pham's model and other existing models in the field of Debugging and Test Theory.

We give real examples with datasets using the new Pham's model.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 41A46

Key Words: four parameters Pham's software reliability model, mean value function, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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1. INTRODUCTION AND PRELIMINARIES

Some software reliability models and studies on their "intrinsic properties", can be found in [2]–[37], [53]–[55].

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by "mean value function" $m(t)$, defined by H. Pham.

The models have been tested with real-world data.

Definition 1. Pham [1] developed the following new mean value function:

$$m(t) = N\left(1 - \frac{\beta}{\beta + at\left(\frac{t}{2}\ln(bt) - \frac{t}{4} + \frac{1}{b}\right)}\right)^{\alpha} \tag{1}$$

where α, β, a, $b > 0$.

Definition 2. The shifted Heaviside step function is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

Definition 3. [40] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

2. MAIN RESULTS

2.1. A NOTE ON THE NEW PHAM'S SOFTWARE RELIABILITY GROWTH MODEL (1)

The investigation of the characteristic "supersaturation" of the model (1) to the horizontal asymptote is important.

Without loosing of generality we will look at the function $m(t)$ with $N = 1$:

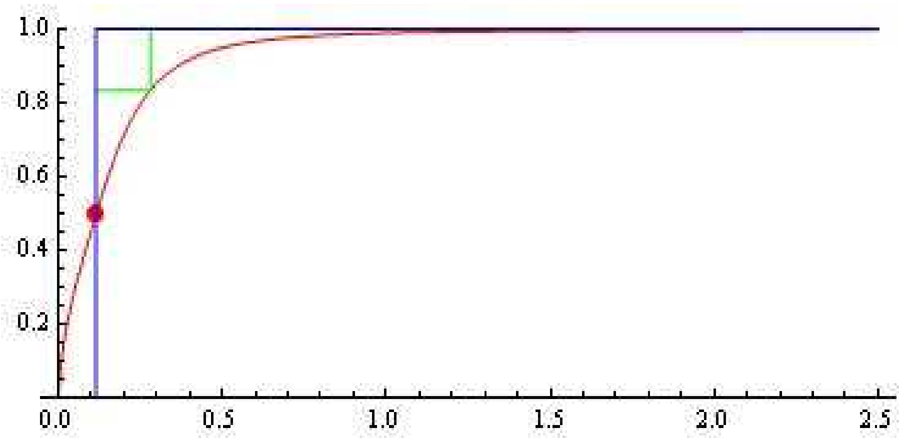

Figure 1: The model (1) for $a = 16$, $b = 10.1$, $\alpha = 0.8$, $\beta = 0.2$ and $t_0 = 0.114987$; H–distance $d = 0.165756$.

Let t_0 is the value for which $m(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the $m(t)$ satisfies the relation

$$m(t_0 + d) = 1 - d. \tag{3}$$

For given α, β, a, $b > 0$ and t_0, the nonlinear equation $m(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The model (1) for $a = 16$, $b = 10.1$, $\alpha = 0.8$, $\beta = 0.2$ and $t_0 = 0.114987$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.165756$.

The model (1) for $a = 16$, $b = 15$, $\alpha = 0.2$, $\beta = 0.1$ and $t_0 = 0.00331237$ is visualized on Fig. 2.

From the nonlinear equation (2) we have: $d = 0.110676$.

The model (1) for $a = 25$, $b = 16$, $\alpha = 0.15$, $\beta = 0.05$ and $t_0 = 0.000322921$ is visualized on Fig. 3.

From the nonlinear equation (2) we have: $d = 0.0682033$.

Some computational examples are presented in Table 1.

2.2. SOME COMPARISONS BETWEEN THE MODEL (1) AND THE SONG, CHANG AND PHAM [2] SOFTWARE GROWTH MODELS

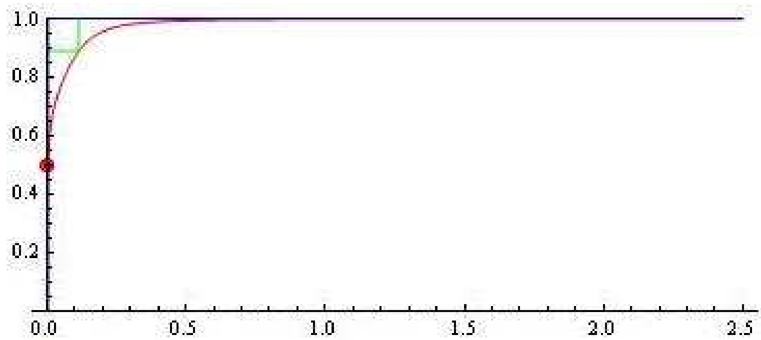

Figure 2: The model (1) for $a = 16$, $b = 15$, $\alpha = 0.2$, $\beta = 0.1$ and $t_0 = 0.00331237$; H–distance $d = 0.110676$.

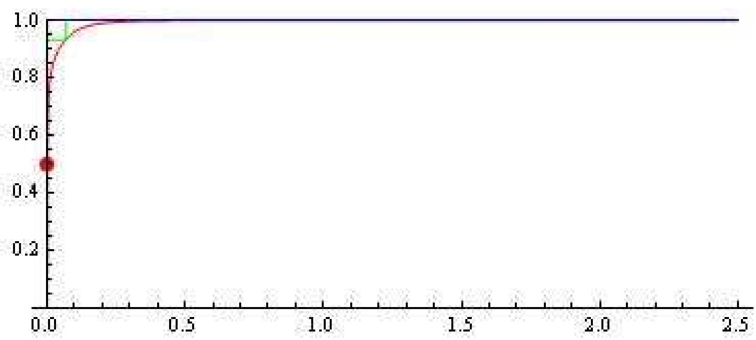

Figure 3: The model (1) for $a = 25$, $b = 16$, $\alpha = 0.15$, $\beta = 0.05$ and $t_0 = 0.000322921$; H–distance $d = 0.0682033$.

a	b	α	β	t_0	$H - distance$
16	0.1	0.8	0.2	0.114987	0.165756
16	15	0.2	0.1	0.00331237	0.110676
25	16	0.15	0.05	0.000322921	0.0682033
25	20	0.1	0.06	0.0000392184	0.0591678
27	22	0.09	0.05	0.0000184578	0.0557664

Table 1: The Hausdorff distance d computed by nonlinear equation (2)

Definition 4. Song, Chang and Pham [2] developed the following software reliability growth models:

$$M(t) = N\left(1 - \frac{\beta}{\beta + \ln \frac{a+e^{bt}}{a+1}}\right)^{\alpha}. \tag{4}$$

and

$$M_1(t) = N\left(1 - \left(\frac{\beta}{\beta + \ln \frac{a+e^{bt}}{a+1}}\right)^{\alpha}\right). \tag{5}$$

where $a, b, \alpha, \beta > 0,\ t > 0$.

A comparison between the models (1) (for $N = 1$), (4) and (5) for fixed parameters $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$ is visualized on Fig. 4.

From the above examples, it can be seen that the "supersaturation" by the $m(t)$ is faster.

Obviously, this "advantage" can actually be used to approximate some specific data.

In the next Section, we will support what is said by analyzing real datasets from other branches of science: population dynamics, biostatistics, and the spread of computer viruses.

2.3. APPLICATIONS

1. We consider the following data "cdf of the number of Bitcoin received per address" (see, [38]:

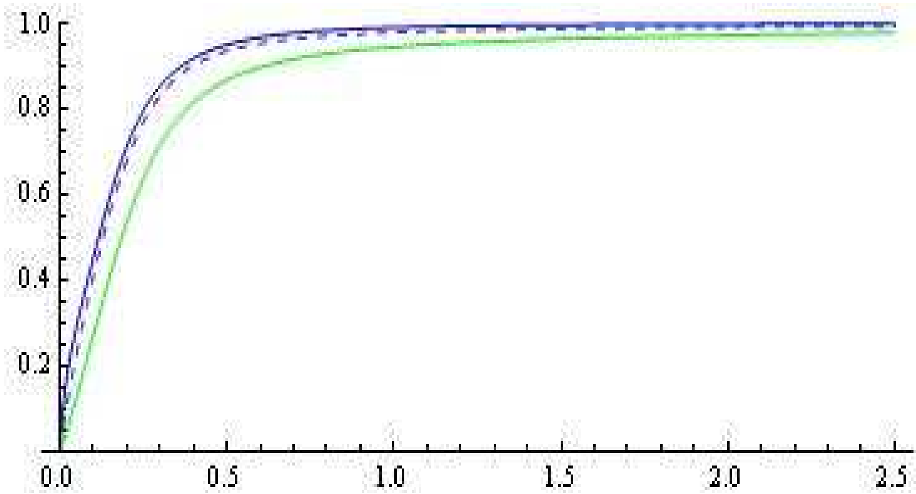

Figure 4: Comparison between the models (1) (blue), (4) (dashed) and (5) (green).

$$data\_CDF\_of\_Bitcoin\_received\_(inransoms)\_per\_address\_in\_C_{CL}$$

$$:= \{\{0.1, 0.0857\}, \{2, 0.1238\}, \{3, 0.6571\}, \{4, 0.6854\}, \{5, 0.8381\},$$

$$\{6, 0.8476\}, \{7, 0.8810\}, \{8, 0.9095\}, \{9, 0.9143\}, \{10, 0.9333\},$$

$$\{12, 0.9429\}, \{14, 0.9571\}, \{18, 0.9667\}, \{20, 0.9762\}, \{23, 0.9810\},$$

$$\{27, 0.9857\}, \{40, 0.9905\}, \{46, 0.9952\}, \{59, 0.9981\}\}.$$

The function $m(t)$ for $a = 15$, $b = 10$, $\beta = 0.04$, $\alpha = 1.29997$ is visualized on Fig. 5.

2. We examine the following data

$$data\_CDF\_of\_ransoms\_received\_per\_address\_in\_C_{CL}$$

$$:= \{\{1, 0.6762\}, \{2, 0.8286\}, \{3, 0.8667\}, \{4, 0.9143\}, \{5, 0.9333\},$$

$$\{6, 0.9429\}, \{7, 0.9524\}, \{8, 0.9571\}, \{9, 0.9667\}, \{10, 0.9714\},$$

$$\{11, 0.9733\}, \{14, 0.9810\}, \{20, 0.9829\}, \{23, 0.9857\}, \{25, 0.9885\},$$

$$\{55, 0.9905\}, \{70, 0.9952\}, \{83, 1\}\}$$

The function $m(t)$ for $a = 0.5$, $b = 0.1$, $\beta = 0.5$, $\alpha = 12.3201$ is visualized on Fig. 6.

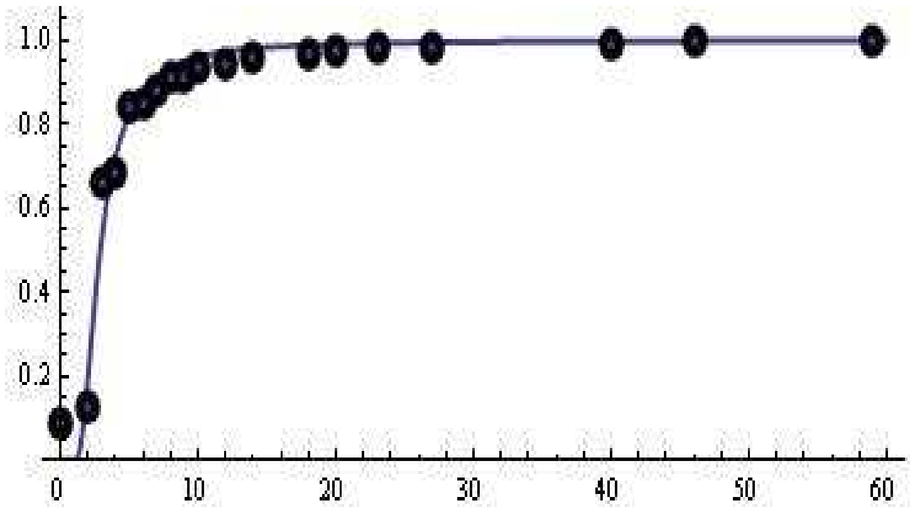

Figure 5: The fitted model.

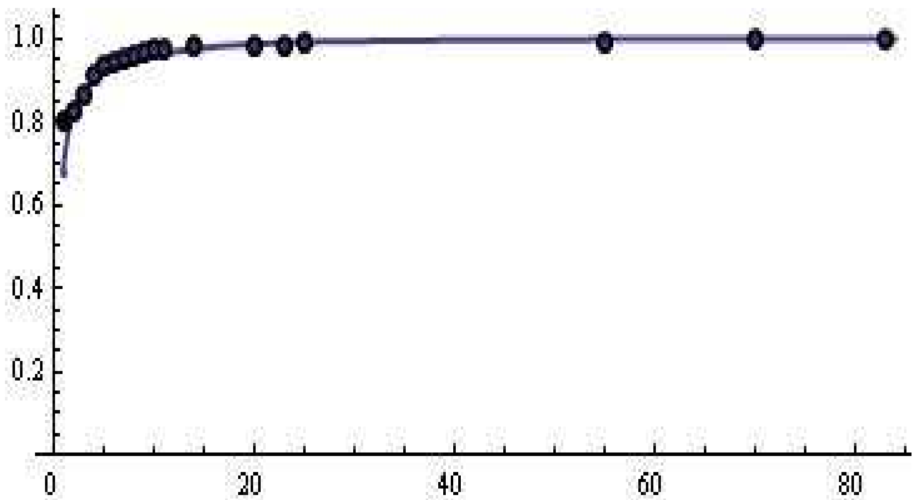

Figure 6: The fitted model.

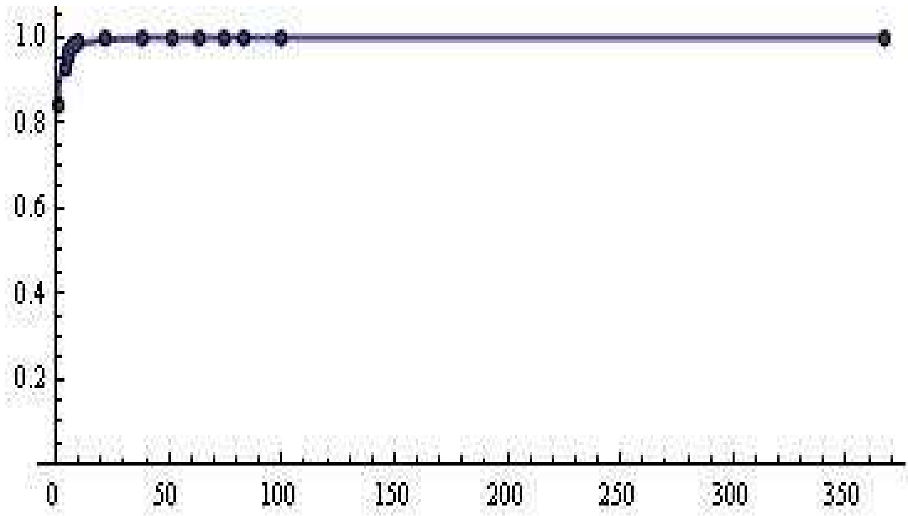

Figure 7: The fitted model.

3. Storm worm one of the most biggest cyber threats of 2008.

We analyze the following data [39]

$$data\_Storm\_IDs := \{\{1, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\},$$
$$\{7, 0.976\}, \{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\},$$
$$\{38, 0.997\}, \{51, 0.998\}, \{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\},$$
$$\{367, 1\}\}$$

The function $m(t)$ for $a = 2$, $b = 0.1$, $\beta = 0.5$, $\alpha = 6.07782$ is visualized on Fig. 7.

4. We examine the following data for the growth of red abalone Haliotis Rufescens in Northern California (see, Fig. 8 [41])

The function $m(t)$ for $a = 0.1084$, $b = 0.43$, $\beta = 1.09111$, $\alpha = 1.28827$ and $N = 194$ is visualized on Fig. 9.

For other approximation and modelling results, see [42]–[52].

3. CONCLUDING REMARKS

We hope that the results will be useful for specialists in this scientific area.

There are some other reliability models that are developed based on the mean value functions $m(t)$ and these can be found in [54]–[55].

Age	*Length(mm)*
1	16.1
2	33.9
3	54.3
4	76.2
5	97.8
6	117.1
7	133.3
8	146.5
9	157.2
10	166
11	173.3
12	179.6
13	185
14	189.9
15	194

Figure 8: The extended data for modeling the growth of red abalone Haliotis Rufescens in Northern California.

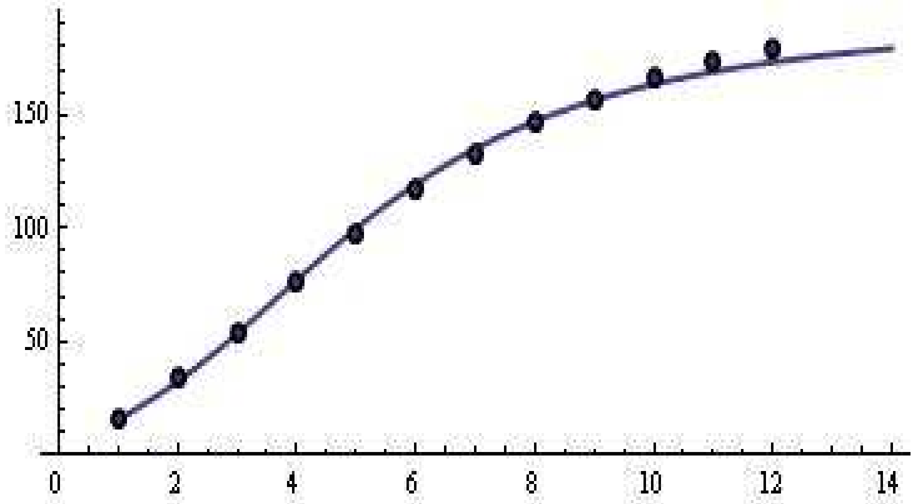

Figure 9: The fitted model (1).

The analysis we conducted in this article on the new Pham's model shows its advantages and reliability compared to other similar models.

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